

# Comment on prescription problem in light-cone gauge

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## Abstract

Recently suggested causal principal value and causal prescriptions for the "spurious singularity" in light-cone gauge theories are nothing but the different guises of usual Mandelstam-Leibbrandt prescription.

Although the light-cone gauge(radiation gauge in light-cone coordinate) is very convenient for the study on QCD vacuum structure and is frequently used in the various branches[1-4] of physics, from time to time there is a debate on the prescription problem for the "spurious singularity" which appears even in the bare propagator of gauge field.

It is proved that if the Cauchy principal value(CPV) prescription for the "spurious singularity"

$$P_{CPV}\left(\frac{1}{k^+}\right) = \frac{1}{2} \left[ \frac{1}{k^+ + i\epsilon} + \frac{1}{k^+ - i\epsilon} \right] \quad (1)$$

is chosen, the calculation of the various Feynman diagrams results in double pole which does not make sense physically.[5] A more reasonable Mandelstam-Leibbrandt(ML) prescription

$$P_{ML}\left(\frac{1}{k^+}\right) = \frac{k^-}{k^+ k^- + i\epsilon} \quad (2)$$

was suggested independently by S.Mandelstam[6] and G.Leibbrandt[7]. Later it was proved in the framework of equal-time canonical quantization that ML prescription preserves the causality[8], and the renormalizability of the gauge theories formulated in this way was discussed[9, 10, 11].

Recently Pimentel and Suzuki suggest two kinds of modified prescriptions for the "spurious singularity", that is causal CPV(CCPV)[12] and causal prescriptions[13]. In this short note I will show that these prescriptions are not new ones but only different forms of ML-prescription.

Their CCPV prescription discussed in Ref.[12] is simply stated as follows: after usual CPV prescription (1) is chosen, in order to recover the causality the constraint that the sign of imaginary part of pole in the "spurious singularity" agrees with that in covariant singularity  $k^2 + i\epsilon = 0$  is required. This

requirement makes change the integration range over the  $k^-$  integral during calculation of various Feynman diagrams and avoid the emergence of double pole singularities. They proved these fact by calculating the following simple integral

$$I = \int \frac{dk}{(k^2 + i\epsilon)[(k - p)^2 + i\epsilon]k^+}. \quad (3)$$

The results of  $I$  for the various prescriptions are

$$\begin{aligned} I_{ML} &= i(-\pi)^{\frac{d}{2}} \frac{(p^2)^{\frac{d}{2}-2}}{p^+} \Gamma(2 - \frac{d}{2}) \\ &\times \left[ \frac{\Gamma(\frac{d}{2} - 2)\Gamma(\frac{d}{2} - 1)}{\Gamma(d - 3)} - \sum_{l=0}^{\infty} \frac{(-1)^l \Gamma(2 - \frac{d}{2} + l)}{l!(\frac{d}{2} - 2 + l)\Gamma(2 - \frac{d}{2})} \left( \frac{\vec{p}_T^2}{p^2} \right)^l \right], \\ I_{CPV} &= i(-\pi)^{\frac{d}{2}} \frac{(p^2)^{\frac{d}{2}-2}}{p^+} \frac{\Gamma(2 - \frac{d}{2})\Gamma(\frac{d}{2} - 2)\Gamma(\frac{d}{2} - 1)}{\Gamma(d - 3)}, \\ I_{CCPV} &= \frac{1}{2} I_{ML}, \end{aligned} \quad (4)$$

where  $d$  is space-time dimensions and  $\vec{p}_T = (p_1, p_2, \dots, p_{d-2})$ . The overall factor  $\frac{1}{2}$  in CCPV prescription is simply explained from the fact that their CCPV prescription is nothing but the half of the usual ML prescription, which can be proved as follows. The usual CPV prescription with covariant singularity is

$$\begin{aligned} \frac{1}{k^2 + i\epsilon} P_{CPV}\left(\frac{1}{k^+}\right) &\equiv \frac{1}{2} \frac{1}{k^2 + i\epsilon} \left( \frac{1}{k^+ + i\delta} + \frac{1}{k^+ - i\delta} \right) \\ &= \frac{1}{4k^-} \frac{1}{k^+ - \frac{\vec{p}_T^2}{2k^-} + i\epsilon\varepsilon(k^-)} \left( \frac{1}{k^+ + i\delta} + \frac{1}{k^+ - i\delta} \right) \end{aligned} \quad (5)$$

where  $\varepsilon(x) = x / |x|$ . The requirement that the sign of imaginary part of pole in the "spurious singularity" agrees with that in covariant one makes the righthand side of Eq.(5) as follows:

$$\frac{1}{4k^-} \frac{1}{k^+ - \frac{\vec{p}_T^2}{2k^-} + i\epsilon\varepsilon(k^-)} \left[ \frac{\theta(k^-)}{k^+ + i\delta} + \frac{\theta(-k^-)}{k^+ - i\delta} \right] \quad (6)$$

where  $\theta(x)$  is usual step function. It is very simple to prove that Eq.(6) is

$$\frac{1}{2} \frac{1}{k^2 + i\epsilon} P_{ML}\left(\frac{1}{k^+}\right)$$

which is the half of usual ML-prescription. Therefore, their CCPV prescription is nothing but the half of the usual ML prescription.

They suggest another modified prescription in Ref.[13] for general non-covariant gauge as follows: for arbitrary vector  $n$ , their causal prescription is

$$\frac{1}{k \cdot n} \rightarrow \frac{\theta(k^0)}{k \cdot n + i\epsilon} + \frac{\theta(-k^0)}{k \cdot n - i\epsilon} \quad (7)$$

which is reduced to

$$\frac{1}{k^+} \rightarrow \frac{\theta(k^0)}{k^+ + i\epsilon} + \frac{\theta(-k^0)}{k^+ - i\epsilon} \quad (8)$$

in the light-cone gauge. Recently they calculated the Wilson-loop by using the prescription (7)[14] and it can be shown easily that their result is in agreement with that of usual ML-prescription in the light cone gauge. However, this is because the prescription (8) is nothing but the ML-prescription.

From the fact

$$P_{ML}\left(\frac{1}{k^+}\right) = P_{CPV}\left(\frac{1}{k^+}\right) - i\pi\delta(k^+) \frac{k^-}{|k^-|}, \quad (9)$$

one can easily show

$$P_{ML}\left(\frac{1}{k^+}\right) = P_{CPV}\left(\frac{1}{k^+}\right) - i\pi\delta(k^+) \frac{\alpha k^+ + k^-}{|\beta k^+ + k^-|} \quad (10)$$

where  $\alpha$  and  $\beta$  are arbitrary c-numbers because of the property of  $\delta(k^+)$ . If one chooses  $\alpha = \beta = 1$ , usual ML prescription coincides with Eq.(8).

Therefore, those prescriptions which were suggested in Ref.[12, 13] are not new ones but only the different forms of usual ML-prescription.

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## **References**

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